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Iterative Construction of CAD Models from Range Images

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Abstract

In the paper we outline a new approach to construction of CAD models from range images. The procedure consists of three steps: range image segmentation, finding an approximate rigid transformation between the current 3-D description and the range image based on segmentation, and range image registration. The object domain is restricted to rigid objects that can be modeled as a composition of non-deformed superquadrics in a simple space occupancy manner. Initial experimental results of range image segmentation are presented to assess the stability of segmentation obtained for range images taken from different viewpoints.

1 Introduction

The primary motivation for this work is to develop a novel method of constructing CAD models from range data, that does not require calibrated positioning of sensor setup or calibrated manipulation of the objects to obtain a complete 3-D description. Instead we relay on the object part structure to sufficiently constrain the matching and the registration of range images from different viewpoints. The object domain is restricted to objects that can be modeled as a composition of non-deformed superquadrics in a simple space occupancy manner. The method is based on integration and adaptation of existing methods for range image segmentation and registration. A direct application of this method is an interactive iterative construction of CAD models, where an operator manually places the object in front of a range scanner in different orientations to incorporate additional data into a current 3-D description of the object. Visualisation of the current 3-D description helps the operator to decide about the appropriate object orientation for the next range image acquisition.

Range image acquisition by 3-D imaging sensors inherently produces only partial 3-D description from a single viewpoint. This description represents a distance between an image plane and points on the object surface in such a way that for each point in the plane there is at most one corresponding point on the object surface. Because of this restriction such description is often referred to as 2.5-D data. Throughout this paper we will use the term *range image* to refer to this 2.5-D data and *3-D description* to refer to a set of 3-D points from the object surface. It is quite obvious that each range image is also a 3-D description but the reverse is generally not true.

Evidently, the main reason for partial description is self-occlusion. Besides, geometry of sensor setup might further reduce the space where 3-D data can be deduced (see [8] for an example of such sensor setup). In order to produce a complete 3-D description, that is a set of densely sampled 3-D points from the object surface, we have to merge the range images obtained from several viewpoints. The later implies either object manipulation or change of sensor position to achieve a relative change of position and orientation of the object with respect to the sensor. To merge data from different viewpoints into a single coordinate frame, we have to find a rigid transformation between the coordinate frames of range images. We can seldom rely only on a calibrated sensor setup, so even if we have an estimate of a rigid transformation we should not merge the data blindly but still perform a data driven process of range image registration by using local shape matching. A rough estimate of transformation obtained during the object manipulation or sensor repositioning can be used as an initial approximation for iterative registration algorithms like the one described in [2]. In our approach initial transformation matrix is calculated on the basis of the moments of inertia of the object and part correspondence.

Of course a complete 3-D description of an object in terms of surface points is not a suitable representation for CAD systems. Further data reduction has to be achieved by partitioning the points into separate parts in the process of segmentation. We selected superellipsoids, a subset of superquadrics [1], as parametric volumetric primitives used in segmen-

tation, because they can model quite a rich set of shapes with a relatively small number of parameters and because of their apparent correspondence to intuitive notion of parts [9].

The paper is organized as follows. First we briefly review the related work. In the section 3 we describe the range image segmentation to obtain part description of the object range image and current 3-D description. The results of segmentation of range images of a real object are presented. We then introduce our method for computing the approximation of the rigid transformation. Finally, we describe how we plan to adapt the iterative registration method to our 3-D data representation.

2 Related Work

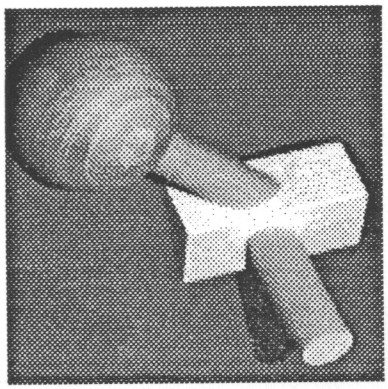
The work that most closely resembles our approach of constructing CAD models was done by Potmesil [10]. He used bicubic surface patches organized into a hierarchical quadtree representation to model the object surface. The rigid transformation between two representations was found by a heuristic search which minimized the positional, orientational, and curvature differences. The results shown in his paper seem to be very good. Unfortunately, only a single example of a "partless" smooth object is presented.

Local shape registration doesn't work for symmetric objects. This problem motivated approaches to construction of 3-D object models that strongly relay on calibrated manipulation of object or sensor to integrate the obtained data from different viewpoints. The method developed by Stenstrom and Connolly [11] uses range and grey level images from different viewpoints to construct 3-D occupancy models. Though the method does not suffer from the problems of ambiguity it is still not suitable for generating 3-D CAD models of symmetrical object, since it may require a very large number of views to obtain an accurate object model from occluding contours.

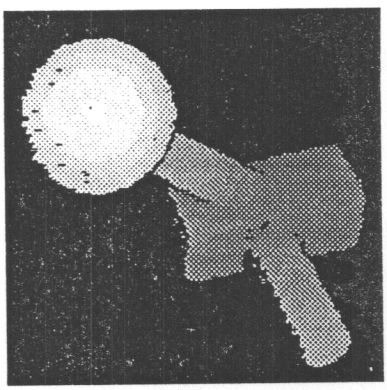
3 Range Image Segmentation

Recent attempts to formalize the problem of image segmentation as a problem of finding a minimum description length encoding of an image in terms of primitives [5] led to quite general and successful algorithms that can segment the range image in terms of volumetric models directly [7] without previous segmentation in terms of surface models [6] that are latter merged into volumetric models [3]. In our experiments we used superellipsoids as volumetric models and tried to find a minimum description length encoding of a range image in terms of these. Informally, the volumetric models produced by segmentation process represent a decomposition of the object into its parts.

To experimentally verify the validity of segmentation algorithm, we used a real range image of a wooden object depicted in Figure 1. The produced list of



a)



b)

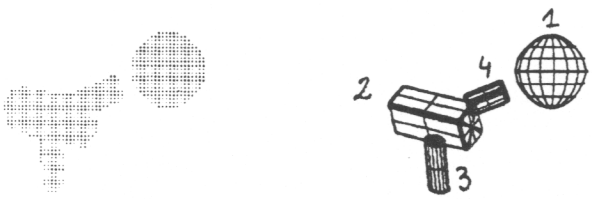
Figure 1: a) Intensity image of the object b) Range image of the object

superellipsoids was then used as an input for a range scanner software simulator. The CAD model was rotated about the Y axis for 45° and 135° and corresponding range images were generated. Note that since the model was produced from a single range image, the sphere and the cylinder in Figure 3 are not connected !

Experiments confirmed that recovered parameters of superquadric models from a single viewpoint range image are not a very reliable description of the object parts [12]. But although the recovered parameters are not reliable, we can conclude from the results of experiments that the decomposition of an object into its parts is reliable since it is based on the object structure. Also the model represents the scanned surface with sufficient precision. For example, one of the size parameters of the sphere in Figure 2 is significantly different from the others, meaning that the part as a whole was not reconstructed correctly, nevertheless the surface of the half sphere is modeled with sufficient precision.

Part no.	a_1	a_2	a_3	ϵ_1	ϵ_2
1	50	39	51	1.1	1.0
2	23	27	36	0.1	0.3
3	19	13	50	0.1	1.0

Figure 2: Original CAD model was rotated about the Y axis for 45° and a synthetic range image was produced (left). The segmentation result is presented on the right.



Part no.	a_1	a_2	a_3	ϵ_1	ϵ_2
1	50	39	51	1.1	1.0
2	22	28	60	0.1	0.2
3	15	10	44	0.1	1.0
4	12	16	28	0.2	1.3

Figure 3: Original CAD model was rotated about the Y axis for 135° and a synthetic range image was produced (left). The segmentation result is presented on the right.

The overall error of surface representation may be larger due to relatively unconstrained growth of models in the segmentation algorithm. We propose the following two additional constraints to be used during the growing stage:

- *Connectedness constraint* - Ensure that the points from the surface of a single superellipsoid form a connected region on the surface itself. The constraint is equivalent to requirement that the parameters (ω, η) corresponding to surface points form a connected region in (ω, η) space.
- *Simple visibility constraint* - Ensure that the points from the surface of a single superellipsoid can actually be "seen" by the scanner. We use the term simple, because we do not account for occlusion caused by another superellipsoid, but only for self occlusion. This constraint divides the rectangle in parameter space (ω, η) into the "visible" and "invisible" region. Visibility is determined by the sign of a dot product of a superellipsoid surface normal and viewing direction.

The first constraint can be easily enforced by using an accumulator array in the (ω, η) space of each superellipsoid. Whenever a point is checked to be added to the model, we check the value at the corresponding (ω, η) coordinates in the accumulator array. If the accumulator value is greater than 0 then the point is connected to the current surface patch, otherwise check the neighbourhood in the parameter space for connectedness. For each point added to the model corresponding accumulator value is incremented.

The second constraint leads no nothing more that a simple comparison of parameter η with the calculated parameter $\eta(\omega)$, evaluated at given ω. The border between the visible and invisible region is generated by a simple equation [4].

Also at a certain stage of model growth it might be useful to use an informed search for the points that are to be added to the model. We do not just incorporate the points that are close to the model, visible, and connected. Instead we calculate the sensitivity of the model error function with respect to the parameters a_1, a_2, and a_3 and then adjust the parameters of expanded model according to the sensitivity of the model. The lower the sensitivity the greater increase in corresponding size parameters.

4 Finding Initial Approximation for a Rigid Transformation of the Object

Given two sets of superellipsoids each representing the reconstructed object parts from range images taken from different viewpoints, we would like to find approximation of the rigid transformation between the two range images. Our method is based on the assumption that the object is not symmetrical, that is the directions of principal axis of inertial moments are well defined. From the analytical expressions for the volume of the superellipsoid [4] it is possible to calculate the center of mass of the object. From the expressions for the inertial moments of a superelipsoids (see Appendices), we can find a coordinate frame with its origin at the center of mass such that the off diagonal inertial moments vanish. Once we determine such frame for each of the two objects, we are left

with four possibilities for the transformation matrix:

- rigid transformation,
- rigid transformation and rotation for 180° about the X axis,
- rigid transformation and rotation for 180° about the Y axis,
- rigid transformation and rotation for 180° about the Z axis,

Simple test based on the part correspondence can determine the right choice for the transformation.

5 Range Image Registration

A current partial 3-D description consists of a set of 3-D data points obtained so far and its representation as a set of volumetric models. Note that the parts of the surfaces of volumetric models that correspond to no data points are just a hypothesis. Each superquadric model also encodes the information about the parameter space of ω and η that has been "covered" by the data points. So a complete 3-D description of a part corresponding to a superquadric "covers" the whole parameter space of ω and η.

Since we require a data driven image registration procedure, the data obtained from the next viewpoint will have to include the regions of points from the volumetric models that have been reliably recovered from data points and are not just hypothesized in the current description. Thus to obtain a reliable estimate of the rigid transformation we wish that data set obtained from the next viewpoint would be a subset of the points incorporated into a current description. Of course such data would only produce a reliable estimate of transformation but add nothing to build a complete 3-D description. On the other hand one might want to maximize the information from the next view trying to reduce the number of necessary viewpoints to form a complete description [8]. But that might lead to a very unreliable estimate of the rigid transformation. Obviously, a trade-off between the contradictory requirements must be found for this local shape matching to work in practice.

The obtained transformation is used as an initial estimate for the modified iterative closest point algorithm [2] that accounts for uncertainty of volumetric models in current 3-D description in the areas with no data points. The distance between the closest point on the superquadric and the data point is weighted according to the uncertainty of the point on the superquadric. That is, if the closest point on the superquadric is just a hypothesized point of the current description, the actual distance between the points is weighted less in the error metric.

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A Inertial Moments of a Superellipsis

A superellipsis is a parameterized plane curve, defined as:

$$\begin{aligned} x(\omega) &= a\cos^{\epsilon_2}\omega \qquad -\pi \le \omega < \pi \\ y(\omega) &= b\sin^{\epsilon_2}\omega \end{aligned} \quad . \tag{1}$$

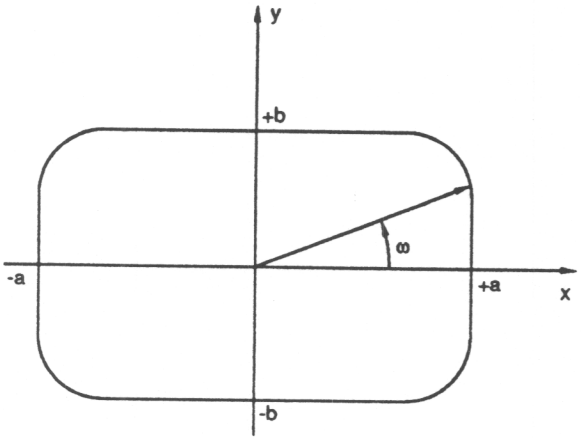

Figure 4: Graph of a superellipsis

The easiest way to calculate the moments of inertia is to introduce superellipsoidal coordinate system with coordiantes r and ω instead of x and y

$$\begin{aligned} x &= ar\cos^{\epsilon_2}\omega \\ y &= br\sin^{\epsilon_2}\omega \end{aligned} \tag{2}$$

The determinant of Jacobian matrix equals

$$|J| = abr\epsilon_2 \sin^{\epsilon_2-1}\omega \cos^{\epsilon_2-1}\omega. \tag{3}$$

and the moments of a superellipsis are as follows

$$\begin{aligned} I_x^0 &= \int\int_S y^2 dxdy \\ &= \int_{-\pi}^{\pi}\int_0^1 b^2r^2\sin^{2\epsilon_2}\omega|J|drd\omega \\ &= \frac{1}{2}ab^3\epsilon_2 B\left(\frac{3\epsilon_2}{2},\frac{\epsilon_2}{2}\right) \end{aligned} \tag{4}$$

$$\begin{aligned} I_y^0 &= \int\int_S x^2 dxdy \\ &= \int_{-\pi}^{\pi}\int_0^1 a^2r^2\cos^{2\epsilon_2}\omega|J|drd\omega \\ &= \frac{1}{2}a^3b\epsilon_2 B\left(\frac{\epsilon_2}{2},\frac{3\epsilon_2}{2}\right) \end{aligned} \tag{5}$$

$$\begin{aligned} I_z^0 &= \int\int_S (x^2+y^2)dxdy \\ &= I_x + I_y \\ &= ab(a^2+b^2)\epsilon_2 B\left(\frac{\epsilon_2}{2},\frac{3\epsilon_2}{2}\right) \end{aligned} \tag{6}$$

where $B(x,y)$ is a beta function.

B Inertial Moments of a Superellipsoid

A surface of a superellipsoid is defined by a vector function of parameters ω and η in the domain $-\pi \le \omega < \pi$ and $-\pi/2 \le \eta \le \pi/2$

$$\mathbf{r}(\eta,\omega) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1\cos^{\epsilon_1}\eta\cos^{\epsilon_2}\omega \\ a_2\cos^{\epsilon_1}\eta\sin^{\epsilon_2}\omega \\ a_3\sin^{\epsilon_1}\eta \end{bmatrix} \tag{7}$$

By using Steiner's rule the inertial moment of a superellipsoid can be determined:

$$\begin{aligned} I_x &= \int\int\int_V (y^2+z^2)dxdydz \\ &= \int_{-a_3}^{+a3}(I_x^0(z)+z^2A(z))dz \end{aligned} \tag{8}$$

$$\begin{aligned} I_y &= \int\int\int_V (x^2+z^2)dxdydz \\ &= \int_{-a_3}^{+a3}(I_y^0(z)+z^2A(z))dz \end{aligned} \tag{9}$$

$$\begin{aligned} I_z &= \int\int\int_V (x^2+y^2)dxdydz \\ &= \int_{-a_3}^{+a3} I_z^0(z)dz \end{aligned} \tag{10}$$

where $A(z)$ is the area of the superellipsis slice with thickness dz

$$A = 2ab\epsilon_2 B\left(\frac{\epsilon_2}{2},\frac{\epsilon_2+2}{2}\right) \tag{11}$$